SADLER UNIT 3 CHAPTER 2

EXERCISE 2A

Q1 a) $z = 4-3i$

$|z| = \sqrt{16+9}$

$= 5$

b) $z = 12+5i$

$|z| = \sqrt{144+25}$

$= 13$

c) $z = 3+2i$

$|z| = \sqrt{9+4}$

$= \sqrt{13}$

d) $z = 3-2i$

$|z| = \sqrt{9+4}$

$= \sqrt{13}$

e) $z = 1+5i$

$|z| = \sqrt{1+25}$

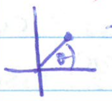
$= \sqrt{26}$

f) $z = 5i$

$|z| = \sqrt{25}$

$= 5$

Q2 a) $z = 2+2i$

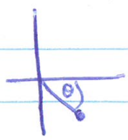


$\tan\theta = \frac{2}{2}$

$\theta = \tan^{-1}(1)$

$\therefore \arg(z) = \frac{\pi}{4}$

b) $z = 2-2i$

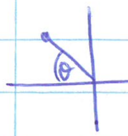


$\tan\theta = \frac{2}{2}$

$\theta = \tan^{-1}(1)$

$\therefore \arg(z) = -\frac{\pi}{4}$

c) $z = -2+2i$



$\tan\theta = \frac{2}{2}$

$\theta = \tan^{-1}(1)$

$\therefore \arg(z) = \frac{3\pi}{4}$

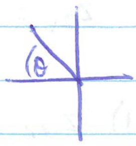
d) $z = -2-2i$

$\tan\theta = \frac{2}{2}$

$\theta = \tan^{-1}(1)$

$\therefore \arg(z) = -\frac{3\pi}{4}$

e) $z = -2+2\sqrt{3}i$

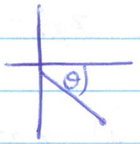


$\tan\theta = \frac{2\sqrt{3}}{2}$

$\theta = \tan^{-1}(\sqrt{3})$

$\therefore \arg(z) = \frac{2\pi}{3}$

f) $z = 3-3\sqrt{3}i$



$\tan\theta = \frac{3\sqrt{3}}{3}$

$\theta = \tan^{-1}(\sqrt{3})$

$\therefore \arg(z) = -\frac{\pi}{3}$

Q3 a) $z_1 = 3\operatorname{cis}\left(\frac{13\pi}{6}\right)$

$= 3\operatorname{cis}\left(\frac{\pi}{6}\right)$

b) $z_2 = 3\operatorname{cis}(3\pi)$

$= 3\operatorname{cis}(\pi)$

c) $z_3 = 4\operatorname{cis}\left(\frac{5\pi}{4}\right)$

$= 4\operatorname{cis}\left(-\frac{3\pi}{4}\right)$

d) $z_4 = 2\operatorname{cis}(-\pi)$

$= 2\operatorname{cis}(\pi)$

e) $z_5 = 6\operatorname{cis}(1)$

f) $z_6 = 5\operatorname{cis}\left(\frac{3\pi}{4}\right)$

g) $z_7 = 8\operatorname{cis}\left(-\frac{5\pi}{6}\right)$

h) $z_8 = 5\operatorname{cis}\left(-\frac{\pi}{2}\right)$

i) $z_9 = 6\operatorname{cis}(2)$

j) $z_{10} = 4\operatorname{cis}(\pi)$

k) $z_{11} = 5\operatorname{cis}\left(-\frac{3\pi}{4}\right)$

l) $z_{12} = 7\operatorname{cis}\left(-\frac{\pi}{6}\right)$

Q4 a) $z_{13} = 5+5i$

$|z_{13}| = \sqrt{25+25}$

$= 5\sqrt{2}$

$\arg(z_{13}) = \tan^{-1}\left(\frac{5}{5}\right)$

$= \frac{\pi}{4}$

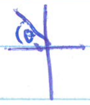
$\therefore z_{13} = 5\sqrt{2}\operatorname{cis}\left(\frac{\pi}{4}\right)$

b) $z_{14} = -3+4i$

$|z_{14}| = \sqrt{9+16}$

$= 5$

$\tan\theta = \frac{4}{3}$



$\theta = \tan^{-1}\left(\frac{4}{3}\right)$

$\theta = 0.9273$

$\therefore \arg(z_{14}) = \pi - 0.9273$

$= 2.2143$

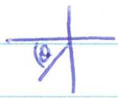
$\therefore z_{14} = 5\operatorname{cis}(2.2143)$

c) $z_{15} = -4-5i$

$|z_{15}| = \sqrt{16+25}$

$= \sqrt{41}$

$\tan\theta = \frac{5}{4}$



$\theta = \tan^{-1}\left(\frac{5}{4}\right)$

$\theta = 0.8961$

$\therefore \arg(z_{15}) = -(\pi - 0.8961)$

$= -2.2455$

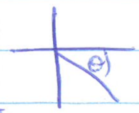
$z_{15} = \sqrt{41}\operatorname{cis}(-2.2455)$

d) $z_{16} = 5-5i$

$|z_{16}| = \sqrt{25+25}$

$= 5\sqrt{2}$

$\tan\theta = \frac{5}{5}$



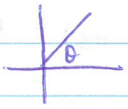
$\arg(z_{16}) = -\frac{\pi}{4}$

$\therefore z_{16} = 5\sqrt{2}\operatorname{cis}\left(-\frac{\pi}{4}\right)$

d) $z_{17} = 5 + 12i$

$|z_{17}| = \sqrt{25+144}$

$= \underline{13}$



$\tan\theta = \frac{12}{5}$

$\theta = \tan^{-1}\left(\frac{12}{5}\right)$

$\therefore \arg(z_{17}) = 1.1760$

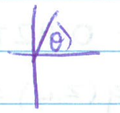
$\therefore z_{17} = \underline{\underline{13 \operatorname{cis}}}(1.1760)$

e) $z_{18} = 1 + 7i$

$|z_{18}| = \sqrt{1+49}$

$= \sqrt{50}$

$= \underline{\underline{5\sqrt{2}}}$



$\tan\theta = 7$

$\theta = \tan^{-1}(7)$

$\arg(z_{18}) = 1.4289$

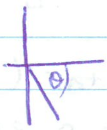
$\therefore z_{18} = \underline{\underline{5\sqrt{2} \operatorname{cis}}}(1.4289)$

f) $z_{19} = 1 - 7i$

$|z_{19}| = \sqrt{1+49}$

$= \sqrt{50}$

$= \underline{\underline{5\sqrt{2}}}$



$\tan\theta = 7$

$\theta = \tan^{-1}(7)$

$\arg(z_{19}) = -1.4289$

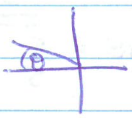
$\therefore z_{19} = \underline{\underline{5\sqrt{2} \operatorname{cis}}}(-1.4289)$

g) $z_{20} = -7 + i$

$|z_{20}| = \sqrt{49+1}$

$= \sqrt{50}$

$= \underline{5\sqrt{2}}$



$\tan\theta = \frac{1}{7}$

$\theta = \tan^{-1}\left(\frac{1}{7}\right)$

$= 0.1419$

$\therefore \arg(z_{20}) = \pi - 0.1419$

$= 2.9997$

$\therefore z_{20} = \underline{5\sqrt{2} \operatorname{cis}}(2.9997)$

h) $z_{21} = 5\sqrt{3} + 5i$

$|z_{21}| = \sqrt{75+25}$

$= \underline{10}$

$\tan\theta = \frac{5}{5\sqrt{3}}$

$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$= \frac{\pi}{6}$

$\therefore z_{21} = \underline{\underline{10 \operatorname{cis}\left(\frac{\pi}{6}\right)}}$

i) $z_{22} = 4i$

$|z_{22}| = 4$

$\theta = \frac{\pi}{2}$

$\therefore z_{22} = \underline{\underline{4 \operatorname{cis}}}\left(\frac{\pi}{2}\right)$

j) $z_{23} = 4$

$|z_{23}| = 4$

$\theta = 0$

$\therefore z_{23} = \underline{\underline{4 \operatorname{cis}(0)}}$

k) $z_{24} = -4$

$|z_{24}| = 4$

$\theta = \pi$

$\therefore z_{24} = \underline{\underline{4 \operatorname{cis}(\pi)}}$

l) $z_{25} = -3i$

$|z_{25}| = 3$

$\theta = -\frac{\pi}{2}$

$\therefore z_{25} = \underline{\underline{3 \operatorname{cis}\left(-\frac{\pi}{2}\right)}}$

m) $z_{26} = 3$

$|z_{26}| = 3$

$\theta = 0$

$\therefore z_{26} = \underline{\underline{3 \operatorname{cis}(0)}}$

Q5 a) $z_{27} = 2 \operatorname{cis}\left(\frac{\pi}{4}\right)$

$= 2\cos\left(\frac{\pi}{4}\right) + i\,2\sin\left(\frac{\pi}{4}\right)$

$= \underline{\underline{\sqrt{2} + \sqrt{2}i}}$

b) $z_{28} = 4 \operatorname{cis}\left(\frac{5\pi}{6}\right)$

$= 4\cos\left(\frac{5\pi}{6}\right) + i\,4\sin\left(\frac{5\pi}{6}\right)$

$= -\frac{4\sqrt{3}}{2} + 2i$

$= \underline{\underline{-2\sqrt{3} + 2i}}$

c) $z_{29} = 4 \operatorname{cis}\left(-\frac{\pi}{3}\right)$

$= 4\cos\left(-\frac{\pi}{3}\right) + i\,4\sin\left(-\frac{\pi}{3}\right)$

$= 2 - \frac{4\sqrt{3}}{2}i$

$= \underline{\underline{2 - 2\sqrt{3}i}}$

d) $z_{30} = 6 \operatorname{cis}\left(\frac{-2\pi}{3}\right)$

$= 6\cos\left(-\frac{2\pi}{3}\right) + i\,6\sin\left(-\frac{2\pi}{3}\right)$

$= -6\left(\frac{1}{2}\right) - \frac{6\sqrt{3}}{2}i$

$= \underline{\underline{-3 - 3\sqrt{3}i}}$

e) $z_{31} = 5 \operatorname{cis}(2\pi)$

$= 5\cos(2\pi) + i\,5\sin(2\pi)$

$= \underline{\underline{5}}$

f) $z_{32} = \operatorname{cis}\left(\frac{7\pi}{2}\right)$

$= \cos\left(\frac{7\pi}{2}\right) + i\sin\left(\frac{7\pi}{2}\right)$

$= \underline{\underline{-i}}$

EXERCISE 2B

Q1 a) $z_1 = 3\,\text{cis}\left(\frac{\pi}{3}\right)$

b) $z_2 = 5\,\text{cis}\left(\frac{2\pi}{3}\right)$

c) $z_3 = 4\,\text{cis}\left(-\frac{5\pi}{6}\right)$

d) $z_4 = 5\,\text{cis}\left(\frac{-\pi}{2}\right)$

e) $z_5 = 4\,\text{cis}(0)$

f) $z_6 = 5\,\text{cis}\left(\frac{\pi}{2}\right)$

g) $z_7 = 5\,\text{cis}\left(\frac{3\pi}{4}\right)$

h) $z_8 = 3\,\text{cis}\left(-\frac{3\pi}{4}\right)$

Q2 $2\cos\left(\frac{\pi}{10}\right) + 2i\sin\left(\frac{\pi}{10}\right)$

$= \underline{2\,\text{cis}\left(\frac{\pi}{10}\right)}$

Q3. $7\cos\left(\frac{5\pi}{8}\right) + 7i\sin\left(\frac{5\pi}{8}\right)$

$= \underline{7\,\text{cis}\left(\frac{5\pi}{8}\right)}$

Q4 $9\cos(30^\circ) + 9i\sin(30^\circ)$

$= \underline{9\,\text{cis}\left(\frac{\pi}{6}\right)}$

Q5 $3\cos(330^\circ) + 3i\sin(330^\circ)$

$= \underline{3\,\text{cis}\left(-\frac{\pi}{6}\right)}$

Q6. $5\cos\left(\frac{3\pi}{2}\right) + 5i\sin\left(\frac{3\pi}{2}\right)$

$= \underline{5\,\text{cis}\left(-\frac{\pi}{2}\right)}$

Q7. $4\cos\left(\frac{8\pi}{3}\right) + 4i\sin\left(\frac{8\pi}{3}\right)$

$= \underline{4\,\text{cis}\left(\frac{2\pi}{3}\right)}$

Q8. $2\cos\left(-\frac{5\pi}{3}\right) + 2i\sin\left(-\frac{5\pi}{3}\right)$

$= \underline{2\,\text{cis}\left(\frac{\pi}{3}\right)}$

Q9. $2\cos(-3\pi) + 2i\sin(-3\pi)$

$= \underline{2\,\text{cis}(\pi)}$

Q10. $7\,\text{cis}\left(\frac{\pi}{2}\right)$

$= 7\cos\frac{\pi}{2} + 7i\sin\frac{\pi}{2}$

$= \underline{7i}$

Q11. $5\,\text{cis}\left(-\frac{\pi}{2}\right)$

$= 5\cos\left(-\frac{\pi}{2}\right) + 5i\sin\left(-\frac{\pi}{2}\right)$

$= \underline{-5i}$

Q12. $\text{cis}\,\pi$

$= \cos\pi + i\sin\pi$

$= \underline{-1}$

Q13. $3\,\text{cis}(2\pi)$

$= 3\cos(2\pi) + 3i\sin(2\pi)$

$= \underline{3}$

Q14. $10\,\text{cis}\left(\frac{\pi}{4}\right)$

$= 10\cos\left(\frac{\pi}{4}\right) + 10i\sin\left(\frac{\pi}{4}\right)$

$= 10\left(\frac{\sqrt{2}}{2}\right) + 10i\left(\frac{\sqrt{2}}{2}\right)$

$= \underline{5\sqrt{2} + 5\sqrt{2}\,i}$

Q15. $4\,\text{cis}\left(\frac{2\pi}{3}\right)$

$= 4\cos\left(\frac{2\pi}{3}\right) + 4i\sin\left(\frac{2\pi}{3}\right)$

$= 4\left(-\frac{1}{2}\right) + 4i\left(\frac{\sqrt{3}}{2}\right)$

$= \underline{-2 + 2\sqrt{3}\,i}$

Q16. $4\,\text{cis}\left(-\frac{2\pi}{3}\right)$

$= 4\cos\left(-\frac{2\pi}{3}\right) + 4i\sin\left(-\frac{2\pi}{3}\right)$

$= 4\left(-\frac{1}{2}\right) + 4i\left(-\frac{\sqrt{3}}{2}\right)$

$= \underline{-2 - 2\sqrt{3}\,i}$

✱ **NOTE:** Complex conjugate to Q15

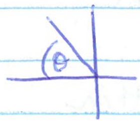
Q17. $12\,\text{cis}\left(-\frac{4\pi}{3}\right)$

$= 12\cos\left(-\frac{4\pi}{3}\right) + 12i\sin\left(-\frac{4\pi}{3}\right)$

$= 12\left(-\frac{1}{2}\right) + 12i\sin\left(\frac{\sqrt{3}}{2}\right)$

$= \underline{-6 + 6\sqrt{3}\,i}$

Q18



Let $z = -7 + 24i$

$r = \sqrt{49 + 576}$

$= \sqrt{625}$

$= 25$

$\tan\theta = \frac{24}{7}$

$\theta = 1.2870$

$\arg(z) = 1.8546$

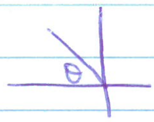
$\therefore\ \underline{z = 25\,\text{cis}(1.8546)}$

Q19. Let $z = -5 + 12i$

$r = |z| = \sqrt{25+144}$

$= \sqrt{169}$

$= 13$



$\tan\theta = \frac{12}{5}$

$\theta = \tan^{-1}\left(\frac{12}{5}\right)$

$= 1.1760$

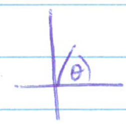
$\therefore \arg(z) = \pi - \theta = \underline{1.9656}$

$\therefore z = \underline{13\,\text{cis}(1.9656)}$

Q20. Let $z = 1 + 2i$

$r = |z| = \sqrt{1+4}$

$= \sqrt{5}$



$\tan\theta = 2$

$\theta = \tan^{-1}(2)$

$= 1.1071$

$\therefore z = \underline{\sqrt{5}\,\text{cis}(1.1071)}$

Q21. $5i$

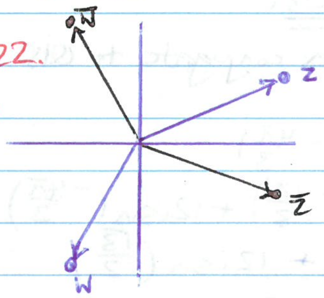
$r = |z| = 5$

$\theta = \frac{\pi}{2}$

$\therefore \underline{5\,\text{cis}\left(\frac{\pi}{2}\right)}$

Q22.

a)



b) if $z = r_1\text{cis}(\alpha)$, then

$\bar{z} = r_1\text{cis}(-\alpha)$

and if $w = r_2\,\text{cis}(\beta)$, then

$\underline{\bar{w} = r_2\,\text{cis}(-\beta)}$.

Q23. $z = 2\text{cis}30° \Rightarrow \bar{z} = 2\text{cis}(-30°)$

Q24 $z = 7\text{cis}120° \Rightarrow \bar{z} = 7\text{cis}(-120°)$

Q25. $z = 4\text{cis}390° \Rightarrow z = 4\text{cis}(30°)$

$\Rightarrow \bar{z} = 4\text{cis}(-30°)$

Q26. $z = 10\text{cis}(-200°) \Rightarrow z = 10\text{cis}(160°)$

$\Rightarrow \underline{\bar{z} = 10\text{cis}(-160°)}$

Q27. $z = 2\text{cis}\left(\frac{\pi}{2}\right) \Rightarrow \bar{z} = 2\text{cis}\left(-\frac{\pi}{2}\right)$

Q28 $z = 5\text{cis}\left(-\frac{3\pi}{4}\right) \Rightarrow \bar{z} = 5\text{cis}\left(\frac{3\pi}{4}\right)$

Q29. $z = 5\text{cis}(0.5) \Rightarrow \bar{z} = 5\text{cis}(-0.5)$

Q30. $z = 5\text{cis}\left(\frac{7\pi}{2}\right) \Rightarrow z = 5\text{cis}\left(-\frac{\pi}{2}\right)$

$\Rightarrow \underline{\bar{z} = 5\text{cis}\left(\frac{\pi}{2}\right)}$

EXERCISE 2C

Q1 $zw = (2+3i)(5-2i)$

$= 10 - 4i + 15i - 6i^2$

$= \underline{16 + 11i}$

Q2. $zw = (3+2i)(-1+2i)$

$= -3 + 6i - 2i + 4i^2$

$= \underline{-7 + 4i}$

Q3. $zw = (3\text{cis}60°)(5\text{cis}20°)$

$= \underline{15\text{cis}(80°)}$.

Q4. $zw = (3\text{cis}120°)(3\text{cis}150°)$

$= 9\text{cis}(270°)$

$= \underline{9\text{cis}(-90°)}$

Q5 $zw = (3\text{cis}30°)(3\text{cis}(-80°))$

$= \underline{9\text{cis}(-50°)}$

Q6. $zw = \left(5\text{cis}\left(\frac{\pi}{3}\right)\right)\left(2\text{cis}\left(\frac{\pi}{4}\right)\right)$

$= 10\text{cis}\left(\frac{4\pi}{12} + \frac{3\pi}{12}\right)$

$= \underline{10\,\text{cis}\left(\frac{7\pi}{12}\right)}$

Q7. $zw = \left(4\operatorname{cis}\left(\frac{\pi}{4}\right)\right)\left(2\operatorname{cis}\left(-\frac{3\pi}{4}\right)\right)$

$= 8\operatorname{cis}\left(-\frac{2\pi}{4}\right)$

$= \underline{\underline{8\operatorname{cis}\left(-\frac{\pi}{2}\right)}}$

Q8. $zw = \left(2\operatorname{cis}(50^\circ)\right)\left(\operatorname{cis}(60^\circ)\right)$

$= 2\operatorname{cis}(110^\circ)$

$= \underline{\underline{2\cos 110^\circ + 2i\sin 110^\circ}}$

Q9. $zw = \left(2\operatorname{cis}(170^\circ)\right)\left(3\operatorname{cis}(150^\circ)\right)$

$= 6\operatorname{cis}(320^\circ)$

$= 6\operatorname{cis}(-40^\circ)$

$= \underline{\underline{6\cos(-40^\circ) + 6i\sin(-40^\circ)}}.$

Q10. $\frac{z}{w} = \frac{6-3i}{3-4i} \times \frac{3+4i}{3+4i}$

$= \frac{18+24i-9i-12i^2}{9+16}$

$= \frac{30+15i}{25}$

$= \underline{\underline{\frac{6}{5} + \frac{3}{5}i}}$

Q11. $\frac{z}{w} = \frac{-6+3i}{-3+4i}$

$= \frac{6-3i}{3-4i}$ (as above).

$= \underline{\underline{\frac{6}{5} + \frac{3}{5}i}}$

Q12. $\frac{z}{w} = \frac{8\operatorname{cis}60^\circ}{2\operatorname{cis}40^\circ}$

$= \underline{\underline{4\operatorname{cis}20^\circ}}$

Q13. $\frac{z}{w} = \frac{5\operatorname{cis}120^\circ}{\operatorname{cis}150^\circ}$

$= \underline{\underline{5\operatorname{cis}(-30^\circ)}}$

Q14. $\frac{z}{w} = \frac{3\operatorname{cis}(-150^\circ)}{3\operatorname{cis}(80^\circ)}$

$= \operatorname{cis}(-230^\circ)$

$= \operatorname{cis}(130^\circ).$ //

Q15 $\frac{z}{w} = \frac{2\operatorname{cis}\left(\frac{3\pi}{5}\right)}{2\operatorname{cis}\left(\frac{2\pi}{5}\right)}$

$= \underline{\underline{\operatorname{cis}\left(\frac{\pi}{5}\right)}}$

Q16. $\frac{z}{w} = \frac{4\operatorname{cis}\left(\frac{\pi}{4}\right)}{2\operatorname{cis}\left(-\frac{3\pi}{4}\right)}$

$= 2\operatorname{cis}\left(\frac{4\pi}{4}\right)$

$= \underline{\underline{2\operatorname{cis}\pi}}$

Q17. $\frac{z}{w} = \frac{5\operatorname{cis}\left(\frac{3\pi}{4}\right)}{2\operatorname{cis}\left(\frac{\pi}{2}\right)}$

$= \frac{5}{2}\operatorname{cis}\left(\frac{3\pi}{4} - \frac{2\pi}{4}\right)$

$= \underline{\underline{\frac{5}{2}\operatorname{cis}\left(\frac{\pi}{4}\right)}}$

Q18. $\frac{z}{w} = \frac{2\operatorname{cis}50^\circ}{5\operatorname{cis}50^\circ}$

$= \underline{\underline{\frac{2}{5}\operatorname{cis}0^\circ}}$

Q19. $zw = 2\operatorname{cis}70^\circ$

$z = 1\operatorname{cis}30^\circ.$

$\therefore w = \frac{2\operatorname{cis}70^\circ}{1\operatorname{cis}30^\circ} = \underline{\underline{2\operatorname{cis}40^\circ}}$

Q20. $zw = 3\operatorname{cis}130^\circ$

$z = 1\operatorname{cis}30^\circ$

$\therefore w = \frac{3\operatorname{cis}130^\circ}{1\operatorname{cis}30^\circ} = \underline{\underline{3\operatorname{cis}100^\circ}}$

Q21. $zw = 2\operatorname{cis}(-60^\circ)$

$z = 1\operatorname{cis}30^\circ.$

$\therefore w = \frac{2\operatorname{cis}(-60^\circ)}{1\operatorname{cis}30^\circ} = \underline{\underline{2\operatorname{cis}(-90^\circ)}}$

Q22. $zw = 2\operatorname{cis}(-130^\circ)$

$z = 1\operatorname{cis}(110^\circ)$

$\therefore w = \frac{2\operatorname{cis}(-130^\circ)}{1\operatorname{cis}(110^\circ)} = 2\operatorname{cis}(-240^\circ)$

$= \underline{\underline{2\operatorname{cis}(120^\circ)}}$

Q23. $zw = 1\text{cis}(-90°)$

$z = 1\text{cis}(110°)$

$\therefore w = \dfrac{1\text{cis}(-90°)}{1\text{cis}(110°)} = 1\text{cis}(-200°)$

$= \underline{\underline{1\text{cis}(160°)}}$

Q24. $zw = 2\text{cis}(-30°)$

$z = 1\text{cis}(110°)$

$\therefore w = \dfrac{2\text{cis}(-30°)}{1\text{cis}(110°)} = \underline{\underline{2\text{cis}(-140°)}}$

Q25 $\dfrac{z}{w} = 2\text{cis}30°$

$z = 2\text{cis}(150°)$

$\therefore w = \dfrac{z}{\left(\frac{z}{w}\right)} = \dfrac{2\text{cis}(150°)}{2\text{cis}(30°)}$

$= \underline{\underline{1\text{cis}(120°)}}$

Q26. $\dfrac{z}{w} = 1\text{cis}70°$

$z = 2\text{cis}150°$

$\therefore w = \dfrac{2\text{cis}150°}{1\text{cis}70°} = \underline{\underline{2\text{cis}80°}}$

Q27. $\dfrac{z}{w} = 1\text{cis}(110°)$

$z = 2\text{cis}(150°)$

$\therefore w = \dfrac{2\text{cis}(150°)}{1\text{cis}(110°)} = \underline{\underline{2\text{cis}40°}}$

Q28. $z = 6\text{cis}40°$, $w = 2\text{cis}30°$

a) $2z = 12\text{cis}40°$

b) $3w = 6\text{cis}30°$

c) $zw = (6\text{cis}40°)(2\text{cis}30°)$

$= \underline{\underline{12\text{cis}(70°)}}$

d) $wz = (2\text{cis}30°)(6\text{cis}40°)$

$= \underline{\underline{12\text{cis}70°}}$

e) $iz = i(6\text{cis}40°)$

$= (1\text{cis}90°)(6\text{cis}40°)$

$= \underline{\underline{6\text{cis}(130°)}}$

f) $iw = i(2\text{cis}30°)$

$= (1\text{cis}90°)(2\text{cis}30°)$

$= \underline{\underline{2\text{cis}(120°)}}$

g) $\dfrac{w}{z} = \dfrac{2\text{cis}30°}{6\text{cis}40°}$

$= \underline{\underline{\frac{1}{3}\text{cis}(-10°)}}$

h) $\dfrac{1}{z} = \dfrac{1}{6\text{cis}40°}$

$= \dfrac{1\text{cis}0°}{6\text{cis}40°}$

$= \underline{\underline{\frac{1}{6}\text{cis}(-40°)}}$

Q29. $z = 8\text{cis}\left(\frac{2\pi}{3}\right)$, $w = 4\text{cis}\left(\frac{3\pi}{4}\right)$

a) $zw = \left(8\text{cis}\left(\frac{2\pi}{3}\right)\right)\left(4\text{cis}\left(\frac{3\pi}{4}\right)\right)$

$= 32\text{cis}\left(\frac{8\pi}{12} + \frac{9\pi}{12}\right)$

$= 32\text{cis}\left(\frac{17\pi}{12}\right)$

$= \underline{\underline{32\text{cis}\left(-\frac{7\pi}{12}\right)}}$

b) $wz = zw = \underline{\underline{32\text{cis}\left(\frac{-7\pi}{12}\right)}}$.

c) $\dfrac{w}{z} = \dfrac{4\text{cis}\left(\frac{3\pi}{4}\right)}{8\text{cis}\left(\frac{2\pi}{3}\right)}$

$= \frac{1}{2}\text{cis}\left(\frac{9\pi}{12} - \frac{8\pi}{12}\right)$

$= \underline{\underline{\frac{1}{2}\text{cis}\left(\frac{\pi}{12}\right)}}$

d) $\dfrac{z}{w} = \dfrac{8\text{cis}\left(\frac{2\pi}{3}\right)}{4\text{cis}\left(\frac{3\pi}{4}\right)}$

$= 2\text{cis}\left(\frac{8\pi}{12} - \frac{9\pi}{12}\right)$

$= \underline{\underline{2\text{cis}\left(\frac{-\pi}{12}\right)}}$

e) $\bar{z} = 8\operatorname{cis}\left(-\frac{2\pi}{3}\right)$

f) $\bar{w} = 4\operatorname{cis}\left(-\frac{3\pi}{4}\right)$

g) $\frac{1}{z} = \frac{1}{8\operatorname{cis}\frac{2\pi}{3}}$

$= \frac{1\operatorname{cis}(0)}{8\operatorname{cis}\left(\frac{2\pi}{3}\right)}$

$= \frac{1}{8}\operatorname{cis}\left(-\frac{2\pi}{3}\right)$

h) $\frac{i}{w} = \frac{i}{4\operatorname{cis}\left(\frac{3\pi}{4}\right)}$

$= \frac{1\operatorname{cis}\left(\frac{\pi}{2}\right)}{4\operatorname{cis}\left(\frac{3\pi}{4}\right)}$

$= \frac{1}{4}\operatorname{cis}\left(\frac{2\pi}{4} - \frac{3\pi}{4}\right)$

$= \frac{1}{4}\operatorname{cis}\left(-\frac{\pi}{4}\right)$

EXERCISE 2D:

Q1. Horizontal line through $\operatorname{Im}(z) = 3$. (D)

Q2 Vertical line through $\operatorname{Re}(z) = 3$ (A)

Q3. All complex numbers at a 45° angle, clockwise. (E)

Q4, All complex numbers at a 135° angle anticlockwise (H)

Q5 All complex numbers within 3 units away from the origin (K)

Q6. $|z - (3-3i)| = 3$

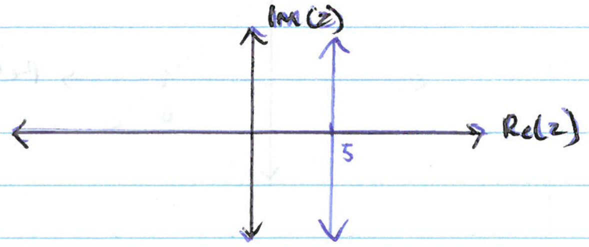
All complex numbers 3 units away from $3-3i$ (L)

Q7. The perpendicular bisector of the line segment joining -8 and $4i$ (M)

Q8 The perpendicular bisector of the line segment joining $-2-3i$ and $4-i$ (P)

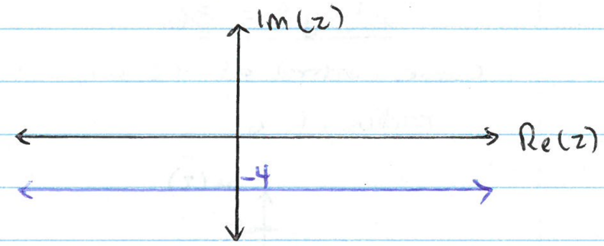
Q9. $\{z : \operatorname{Re}(z) = 5\}$

Let $\operatorname{Re}(z) = x$ $\therefore$ $x = 5$



Q10. $\{z : \operatorname{Im}(z) = -4\}$

Let $\operatorname{Im}(z) = y$ $\therefore$ $y = -4$



Q11. $\{z : \arg(z) = \frac{\pi}{3}\}$

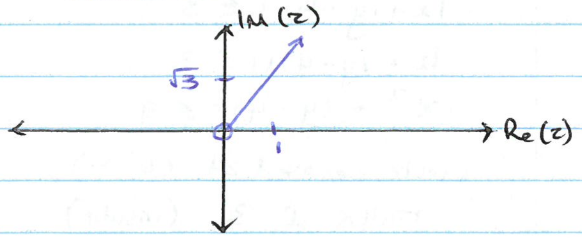
Let $z = x + iy$,

$\therefore \arg(z) = \tan^{-1}\left(\frac{y}{x}\right)$.

$\frac{\pi}{3} = \tan^{-1}\left(\frac{y}{x}\right)$

$y = \tan\frac{\pi}{3}x$, $x > 0$.

$y = \sqrt{3}x$.



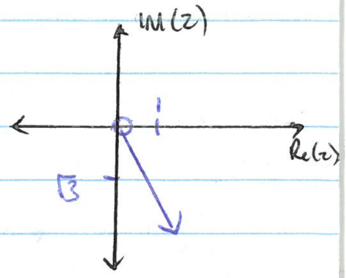
Q12. $\{z : \arg(z) = -\frac{\pi}{3}\}$

$\arg(z) = \tan^{-1}\left(\frac{y}{x}\right)$

$-\frac{\pi}{3} = \tan^{-1}\left(\frac{y}{x}\right)$

$y = \tan\left(-\frac{\pi}{3}\right)x$

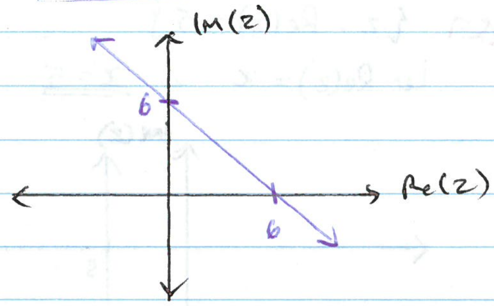
$y = -\sqrt{3}x$



Q13. $\{z : Re(z) + Im(z) = 6\}$

Let $z = x + iy$,

$\therefore$ $\underline{x + y = 6}$.



Q14. $\{z : |z| = 6\}$

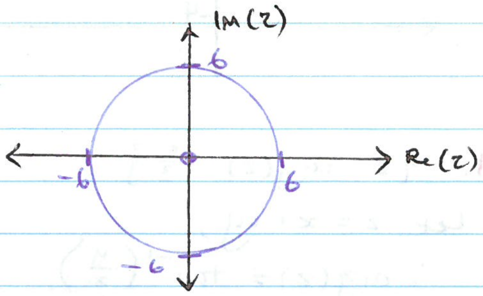
Let $z = x + iy$,

$|z|^2 = x^2 + y^2$

$\underline{x^2 + y^2 = 36}$.

$\therefore$ circle centred at $(0,0)$,

radius of 6.



Q15 $\{z : |z - 4i| \leq 3\}$

Let $z = x + iy$,

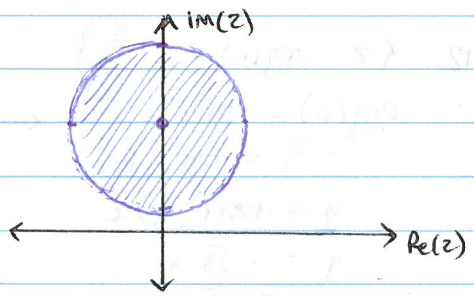
$|x + iy - 4i| \leq 3$.

$|x + (y-4)i| \leq 3$

$x^2 + (y-4)^2 \leq 9$

$\therefore$ circle centred at $(0, 4)$,

radius of 3. (inside)



Q16. $\{z : |z - (2+3i)| = 4\}$

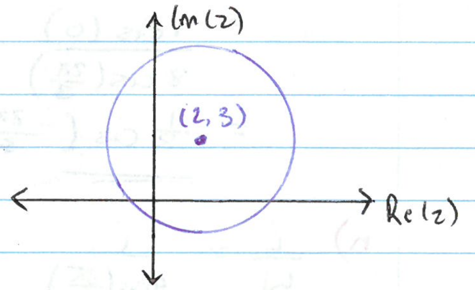
Let $z = x + iy$,

$|x + iy - 2 - 3i| = 4$

$|(x-2) + i(y-3)| = 4$

$(x-2)^2 + (y-3)^2 = 16$

$\therefore$ circle centred at $(2,3)$, radius of 4.



Q17. $\{z : |z - (2-3i)| = 4\}$

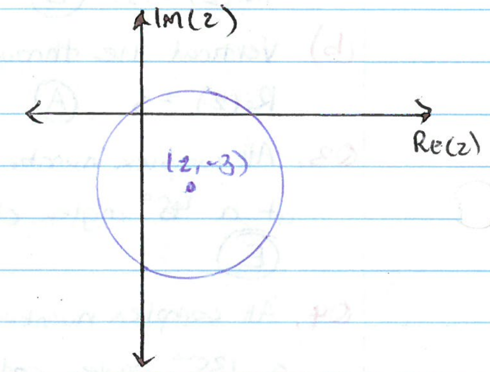
Let $z = x + iy$,

$|x + iy - 2 + 3i| = 4$

$|(x-2) + i(y+3)| = 4$

$(x-2)^2 + (y+3)^2 = 16$

$\therefore$ circle centred at $(2,-3)$, radius 4.



Q18. $\{z : |z-2| = |z-6|\}$

Let $z = x + iy$,

$|x + iy - 2| = |x + iy - 6|$

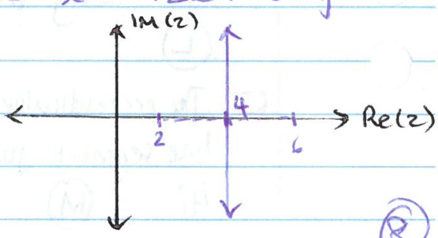
$|(x-2) + iy| = |(x-6) + iy|$

$(x-2)^2 + y^2 = (x-6)^2 + y^2$

$x^2 - 4x + 4 + y^2 = x^2 - 12x + 36 + y^2$

$8x = 32$

$\underline{x = 4}$



Q19 $\{z : |z-6i| = |z-2|\}$

Let $z = x+iy$,

$|x+yi-6i| = |x+iy-2|$

$|x+(y-6)i| = |(x-2)+iy|$

$x^2+(y-6)^2 = (x-2)^2+y^2$

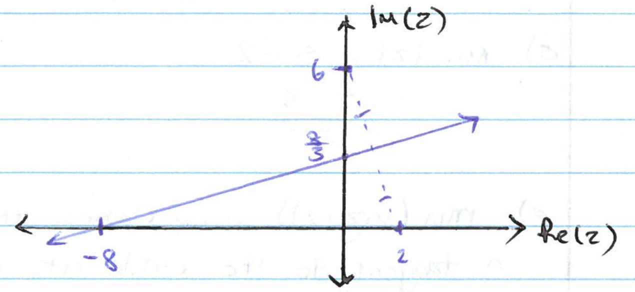
$x^2+y^2-12y+36 = x^2-4x+4+y^2$

$-12y+36 = -4x+4$

$-12y+4x = -32$

$-3y+x = -8$

$\underline{\underline{3y-x = 8}}$



Q20 $\{z : |z-(2+i)| = |z-(4-5i)|\}$

Let $z = x+iy$,

$|x+iy-2-i| = |x+iy-4+5i|$

$|(x-2)+i(y-1)| = |(x-4)+(y+5)|$

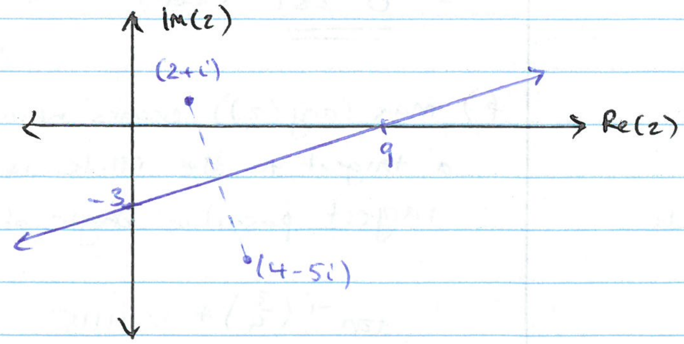
$(x-2)^2+(y-1)^2 = (x-4)^2+(y+5)^2$

$x^2-4x+4+y^2-2y+1 = x^2-8x+16+y^2+10y+25$

$-4x-2y+5 = -8x+10y+41$

$4x-12y = 36$

$\underline{\underline{x-3y = 9}}$

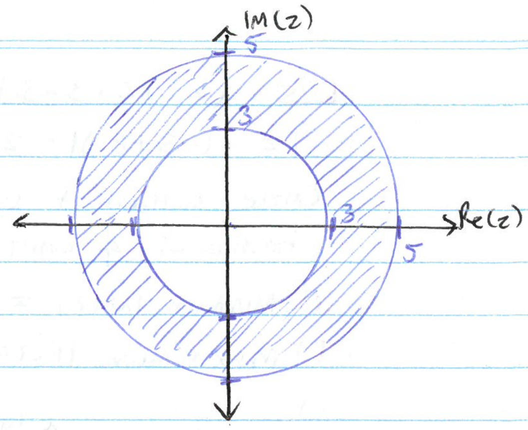


Q21 $\{z : 3 \le |z| \le 5\}$

Let $z = x+iy$,

$3^2 \le x^2+y^2 \le 5^2$

$9 \le x^2+y^2 \le 25$



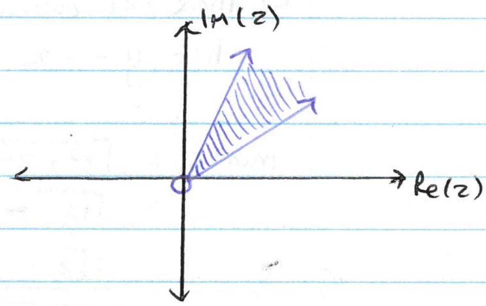
Q22 $\{z : \frac{\pi}{6} \le \arg(z) \le \frac{\pi}{3}\}$

$\frac{\pi}{6} \le \tan^{-1}\left(\frac{y}{x}\right) \le \frac{\pi}{3}$

$\tan\frac{\pi}{6} \le \frac{y}{x} \le \tan\frac{\pi}{3}$

$\frac{1}{\sqrt{3}} \le \frac{y}{x} \le \sqrt{3}$

$\frac{\sqrt{3}}{3}x \le y \le \sqrt{3}x \quad , x>0$



Q23 $\{z : \mathrm{Im}(z) \ge 2\mathrm{Re}(z)+1\}$

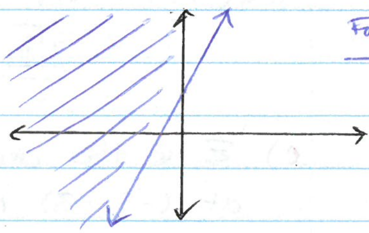
Let $z = x+yi$,

$y \ge 2x+1$

Test $(0,0)$.

$0 \ge 1$

<u>False!</u>



Q24 $\{z : \mathrm{Im}(z) < 2-\mathrm{Re}(z)\}$

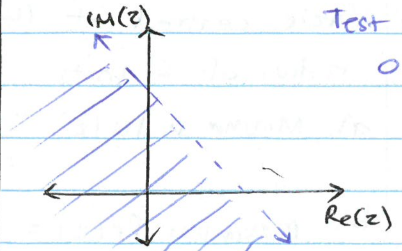
Let $z = x+yi$

$y < 2-x \Rightarrow y+x < 2$

Test $(0,0)$

$0 < 2$

<u>True!</u>



Q25 $\{z : |z+3-3i| = 2\}$

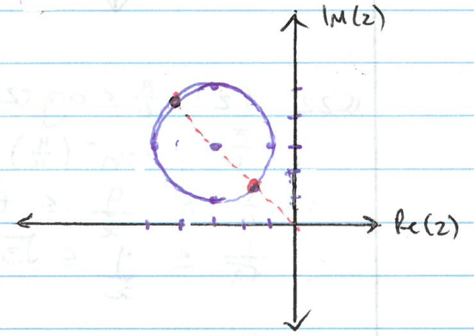
$|z-(-3+3i)| = 2$

Circle, centred at $(-3, 3)$

radius of 2 units

a) Minimum $\text{Im}(z) = 3-2 = 1$

b) maximimum $|\text{Re}(z)| = |-3-2| = \underline{\underline{5}}$



c) Min $|z|$ occurs on the line $y=-x$

$\min|z| = \sqrt{3^2+3^2} - 2$

$= \sqrt{18} - 2$

$= \underline{\underline{3\sqrt{2} - 2}}$ units

d) max $|z|$ also occurs on line $y=-x$

$\max|z| = \sqrt{18} + 2$

$= \underline{\underline{3\sqrt{2}+2}}$ units

e) $\bar{z}$ is the circle centred at $(-3,-3)$ radius 2.

$\therefore \max|\bar{z}| = \max|z|$

$= \underline{\underline{3\sqrt{2}+2}}$ units

Q26. $\{z : |z-(4+3i)| = 2\}$

Circle, centred at $(4,3)$

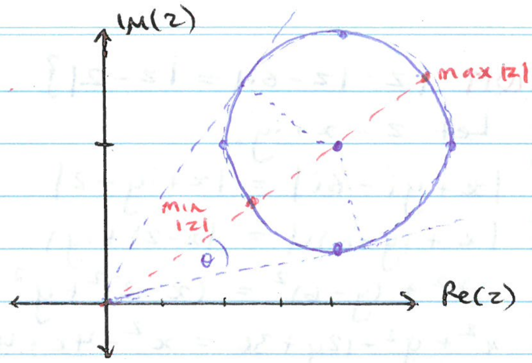
radius of 2 units.

a) minimum $\text{Im}(z) = 3-2$

$= 1$

b) maximum $\text{Re}(z) = 4+2$

$= 6$ //



c) $\max|z| = \sqrt{4^2+3^2} + 2$

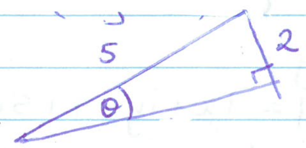
$= 5+2$

$= \underline{\underline{7}}$

d) $\min|z| = 5-2$

$= \underline{\underline{3}}$

e) $\min(\arg(z))$ occurs when there is a tangent to the circle with smallest angle of inclination.



$\sin\theta = \frac{2}{5}$

$\theta = \underline{\underline{0.4115}}$ (4dp)

$\therefore \tan^{-1}\left(\frac{3}{4}\right) - 0.4115$

$= \underline{\underline{0.23^R}}$ (2dp)

f) $\max(\arg(z))$ occurs when there is a tangent to the circle with largest possible angle of inclination

$\therefore \tan^{-1}\left(\frac{3}{4}\right) + 0.4115$

$= \underline{\underline{1.06^R}}$ (2dp)

Q27

$|z-(2+3i)| = 2|z-(5-3i)|$

Let $z = x+yi$,

$|x+yi-2-3i| = 2|x+yi-5+3i|$

$|(x-2)+(y-3)i| = 2|(x-5)+(y+3)i|$

$(x-2)^2 + (y-3)^2 = 4[(x-5)^2 + (y+3)^2]$

$x^2-4x+4 + y^2-6y+9 = 4[x^2-10x+25 + y^2+6y+9]$

$x^2-4x+y^2-6y+13 = 4x^2-40x+4y^2+24y+136$

$3x^2-36x+3y^2+30y = -123$

$x^2-12x+y^2+10y = -41$

$(x-6)^2-36+(y+5)^2-25 = -41$

$(x-6)^2+(y+5)^2 = 20$

$\therefore$ circle centred at $(6,-5)$, radius $\sqrt{20} = \underline{\underline{2\sqrt{5}}}$.

Q28

$|z-(10+5i)| = 3|z-(2-3i)|$

Let $z = x+yi$,

$|x+yi-10-5i| = 3|x+yi-2+3i|$

$|(x-10)+(y-5)i| = 3|(x-2)+(y+3)i|$

$(x-10)^2 + (y-5)^2 = 3[(x-2)^2 + (y+3)^2]$

$x^2-20x+100+y^2-10y+25 = 9[x^2-4x+4+y^2+6y+9]$

$x^2-20x+y^2-10y+125 = 9x^2-36x+9y^2+54y+117$

$8x^2-16x+8y^2+64y = 8$

$x^2-2x+y^2+8y = 1$

$(x-1)^2-1+(y+4)^2-16 = 1$

$(x-1)^2+(y+4)^2 = 18$

$\therefore$ circle centred at $(1,-4)$, radius $\sqrt{18} = \underline{\underline{3\sqrt{2}}}$

EXERCISE 2E

Q1 $z^6 = 1$

Let $1 = 1\text{cis}\,0$

$z^6 = 1\text{cis}\,0$

$z_k = 1\text{cis}\left(\frac{0 + 2\pi k}{6}\right)$

$z_1 = 1\text{cis}\left(\frac{\pi}{3}\right)$

$z_2 = 1\text{cis}\left(\frac{2\pi}{3}\right)$

$z_3 = 1\text{cis}(\pi)$

$z_4 = 1\text{cis}\left(\frac{8\pi}{6}\right)$
$= 1\text{cis}\left(\frac{-2\pi}{3}\right)$

$z_5 = 1\text{cis}\left(\frac{10\pi}{6}\right)$
$= 1\text{cis}\left(\frac{-\pi}{3}\right)$

$z_6 = 1\text{cis}(2\pi)$
$= 1\text{cis}(0)$

Q2. $z^8 = 1$

Let $1 = 1\text{cis}\,0$

$z^8 = 1\text{cis}\,0$

$z_k = 1\text{cis}\left(\frac{0 + 2\pi k}{8}\right)$

$z_1 = 1\text{cis}\left(\frac{\pi}{4}\right) = 1\text{cis}(45°)$

$z_2 = 1\text{cis}\left(\frac{\pi}{2}\right) = 1\text{cis}(90°)$

$z_3 = 1\text{cis}\left(\frac{3\pi}{4}\right) = 1\text{cis}(135°)$

$z_4 = 1\text{cis}(\pi) = 1\text{cis}(180°)$

$z_5 = 1\text{cis}\left(\frac{10\pi}{8}\right)$
$= 1\text{cis}\left(\frac{-3\pi}{4}\right) = 1\text{cis}(-135°)$

$z_6 = 1\text{cis}\left(\frac{12\pi}{8}\right)$
$= 1\text{cis}\left(-\frac{\pi}{2}\right) = 1\text{cis}(-90°)$

$z_7 = 1\text{cis}\left(\frac{14\pi}{8}\right)$
$= 1\text{cis}\left(-\frac{\pi}{4}\right) = 1\text{cis}(-45°)$

$z_8 = 1\text{cis}(2\pi)$
$= 1\text{cis}(0) = 1\text{cis}(0°)$

Q3. $z^7 = 1$

Let $1 = 1\text{cis}\,0$,

$z^7 = 1\text{cis}\,0$

$z_k = 1\text{cis}\left(\frac{0 + 2\pi k}{7}\right)$

$z_1 = 1\text{cis}\left(\frac{2\pi}{7}\right)$

$z_2 = 1\text{cis}\left(\frac{4\pi}{7}\right)$

$z_3 = 1\text{cis}\left(\frac{6\pi}{7}\right)$

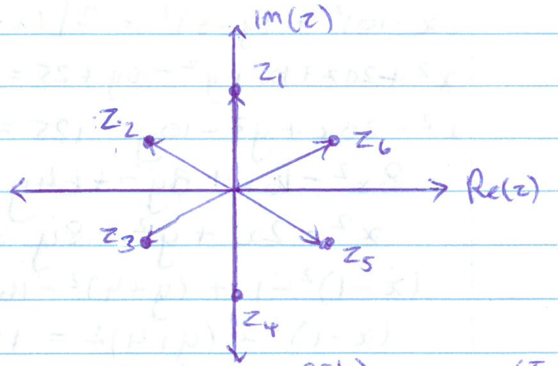
$z_4 = 1\text{cis}\left(\frac{8\pi}{7}\right)$
$= 1\text{cis}\left(\frac{-6\pi}{7}\right)$

$z_5 = 1\text{cis}\left(\frac{10\pi}{7}\right)$
$= 1\text{cis}\left(\frac{-4\pi}{7}\right)$

$z_6 = 1\text{cis}\left(\frac{12\pi}{7}\right)$
$= 1\text{cis}\left(-\frac{2\pi}{7}\right)$

$z_7 = 1\text{cis}(2\pi)$
$= 1\text{cis}(0)$

Q4. $z^6 = -64 = 64\text{cis}(\pi)$



$\therefore z_k = \sqrt[6]{64}\text{cis}\left(\frac{\pi + 2\pi k}{6}\right) = 2\text{cis}\left(\frac{\pi + 2\pi k}{6}\right)$

$z_1 = 2\text{cis}\left(\frac{3\pi}{6}\right)$
$= 2\text{cis}\left(\frac{\pi}{2}\right)$

Then $z_2 = 2\text{cis}\left(\frac{5\pi}{6}\right)$

$z_3 = 2\text{cis}\left(\frac{7\pi}{6}\right) = 2\text{cis}\left(\frac{-5\pi}{6}\right)$

$z_4 = 2\text{cis}\left(\frac{9\pi}{6}\right) = 2\text{cis}\left(-\frac{\pi}{2}\right)$

$z_5 = 2\text{cis}\left(\frac{11\pi}{6}\right) = 2\text{cis}\left(-\frac{\pi}{6}\right)$

$z_6 = 2\text{cis}\left(\frac{13\pi}{6}\right) = 2\text{cis}\left(\frac{\pi}{6}\right)$

Q5 $z^5 = -4+4i$

$= \sqrt{16+16}\operatorname{cis}(135°)$

$= \sqrt{32}\operatorname{cis}(135°)$

$\therefore z_k = \sqrt[5]{32}\operatorname{cis}\left(\frac{135° + 360°k}{5}\right)$

$= 2\operatorname{cis}(27° + 72°k)$

$z_1 = 2\operatorname{cis}(99°)$

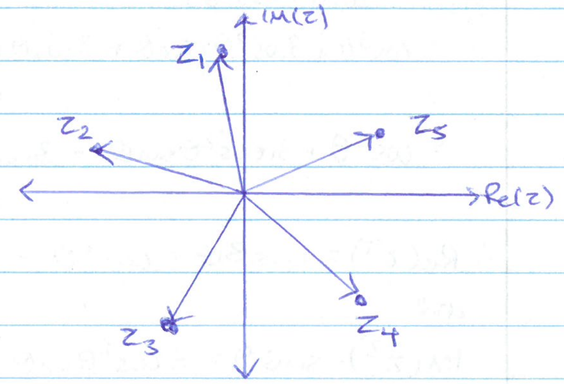
$z_2 = 2\operatorname{cis}(171°)$

$z_3 = 2\operatorname{cis}(243°)$

$= 2\operatorname{cis}(-117°)$

$z_4 = 2\operatorname{cis}(-45°)$

$z_5 = 2\operatorname{cis}(27°)$

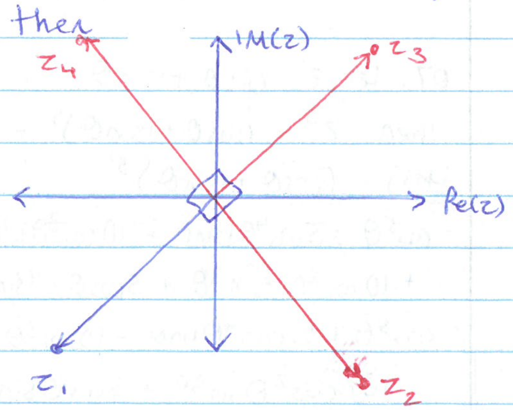


Q6. $z^4 = -119-120i$

If $z_1 = 2+3i$

$\therefore$ and $z_1^4 = -119-120i$,

then



$z_2 = iz_1 = 120-119i$

$z_3 = iz_2 = 119+120i$

$z_4 = iz_3 = -120+119i$

Q7. a) $(2+i)^2$

$= 2^2 + 2(2)(i) + i^2$

$= 4 + 4i - 1$

$= 3+4i$

b) $((2+i)^2)^2$

$= (3+4i)^2$

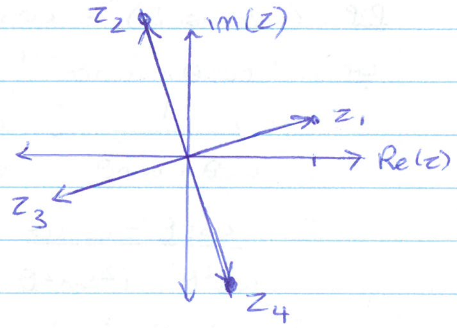
$= 9 + 2(3)(4i) + 16i^2$

$= 9 + 24i - 16$

$= -7+24i$

c) $z^4 = -7+24i$

$\therefore$ Let $z_1 = 2+i$



d) If $z_1 = 2+i$, then

$z_2 = iz_1$

$= 2i-1 = -1+2i$

$z_3 = iz_2$

$= -2-i$

$z_4 = iz_3$

$= 1-2i$

Q8. $z_1 = 2\operatorname{cis}(20°)$

$z_1^5 = 2^5\operatorname{cis}(5(20))$.

$z^5 = 32\operatorname{cis}(100°)$

$\therefore z_k = 2\operatorname{cis}\left(\frac{100+360(k-1)}{5}\right)$

$z_1 = 2\operatorname{cis}(20+72(0))$

$= 2\operatorname{cis}(20°)$

$z_2 = 2\operatorname{cis}(92°)$

$z_3 = 2\operatorname{cis}(164°)$ 20

$z_4 = 2\operatorname{cis}(-124°)$

$z_5 = 2\operatorname{cis}(-52°)$

Q9. $z_1 = 2+4i$

$\therefore z_2 = iz_1$

$= 2i - 4$

$= -4+2i$

$z_3 = iz_2$

$= -2-4i$

$z_4 = iz_3$

$= 4-2i$

EXERCISE 2F

Q1. $\cos\theta + i\sin\theta = \text{cis}\,\theta$

RTP: $(\cos\theta + i\sin\theta)^{-1} = \cos(-\theta) + i\sin(-\theta)$

LHS) $(\cos\theta + i\sin\theta)^{-1}$

$= \dfrac{1}{\cos\theta + i\sin\theta} \times \dfrac{\cos\theta - i\sin\theta}{\cos\theta - i\sin\theta}$

$= \dfrac{\cos\theta - i\sin\theta}{\cos^2\theta - i^2\sin^2\theta}$

$= \dfrac{\cos\theta - i\sin\theta}{1}$

$= \cos\theta - i\sin\theta$

$= \cos(-\theta) + i\sin(-\theta)$

$=$ RHS

Q2 $z = \text{cis}\left(\frac{\pi}{6}\right)$

$\therefore z^4 = 1\,\text{cis}\left(\frac{2\pi}{3}\right)$

Q3. $z = 2\,\text{cis}\left(\frac{\pi}{6}\right)$

$\therefore z^5 = 2^5\,\text{cis}\left(\frac{5\pi}{6}\right)$

$= 32\,\text{cis}\left(\frac{5\pi}{6}\right)$

Q4. $z = 3\,\text{cis}\left(\frac{\pi}{3}\right)$

$\therefore z^5 = 3^5\,\text{cis}\left(\frac{5\pi}{3}\right)$

$= 243\,\text{cis}\left(-\frac{\pi}{3}\right)$

Q5. let $z = \cos\theta + i\sin\theta$

then $z^2 = (\cos\theta + i\sin\theta)^2 = \text{cis}(2\theta)$

LHS) $(\cos\theta + i\sin\theta)^2$

$= \cos^2\theta + 2\cos\theta i\sin\theta + i^2\sin^2\theta$

$= \cos^2\theta - \sin^2\theta + 2i\sin\theta\cos\theta$

$\therefore \text{Re}(z^2) = \cos 2\theta = \cos^2\theta - \sin^2\theta$

and

$\text{Im}(z^2) = \sin 2\theta = 2\sin\theta\cos\theta$

Q6. let $z = \cos\theta + i\sin\theta$

then $z^3 = (\cos\theta + i\sin\theta)^3 = \text{cis}(3\theta)$

LHS) $(\cos\theta + i\sin\theta)^3$

$= \cos^3\theta + 3\cos^2\theta\, i\sin\theta + 3\cos\theta\, i^2\sin^2\theta + i^3\sin^3\theta$

$= \cos^3\theta + 3i\cos^2\theta\sin\theta - 3\cos\theta\sin^2\theta - i\sin^3\theta$

$\therefore \text{Re}(z^3) = \cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta$

and

$\text{Im}(z^3) = \sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta$

$\therefore \cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta$

$= \cos^3\theta - 3\cos\theta(1-\cos^2\theta)$

$= \cos^3\theta - 3\cos\theta + 3\cos^3\theta$

$= 4\cos^3\theta - 3\cos\theta$

Q7. let $z = \cos\theta + i\sin\theta$

then $z^5 = (\cos\theta + i\sin\theta)^5 = \text{cis}(5\theta)$

LHS) $(\cos\theta + i\sin\theta)^5$

$= \cos^5\theta + 5\cos^4\theta\, i\sin\theta + 10\cos^3\theta\, i^2\sin^2\theta + 10\cos^2\theta\, i^3\sin^3\theta + 5\cos\theta\, i^4\sin^4\theta + i^5\sin^5\theta$

$= \cos^5\theta + 5i\cos^4\theta\sin\theta - 10\cos^3\theta\sin^2\theta - 10i\cos^2\theta\sin^3\theta + 5\cos\theta\sin^4\theta + i\sin^5\theta.$

$\therefore \text{Re}(z^5) = \cos(5\theta) = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$

and

$\text{Im}(z^5) = \sin(5\theta) = 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta$

Q8. $(1+i)^6$

Let $z = 1+i$,

$|z| = \sqrt{1+1}$

$= \sqrt{2}$

$\arg(z) = \tan^{-1}\left(\frac{1}{1}\right)$

$= \frac{\pi}{4}$

$\therefore z = \sqrt{2}\,\text{cis}\left(\frac{\pi}{4}\right)$

$\therefore z^6 = (\sqrt{2})^6\,\text{cis}\left(\frac{6\pi}{4}\right)$

$= 8\,\text{cis}\left(-\frac{\pi}{2}\right)$

Q9. $(\sqrt{3}+i)^5$

Let $z = \sqrt{3}+i$

$|z| = \sqrt{3+1}$

$= 2$

$\arg(z) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$= \frac{\pi}{6}$.

$\therefore z = 2\,\text{cis}\left(\frac{\pi}{6}\right)$

$\therefore z^5 = 32\,\text{cis}\left(\frac{5\pi}{6}\right)$

Q10. $(-3+3\sqrt{3}i)^4$

Let $z = -3+3\sqrt{3}i$

$|z| = \sqrt{9+27}$

$= 6$.

$\theta = \tan^{-1}\left(\frac{3\sqrt{3}}{3}\right)$

$= \frac{\pi}{3}$

$\therefore \arg(z) = \frac{2\pi}{3}$.

$\therefore z = 6\,\text{cis}\left(\frac{2\pi}{3}\right)$

$z^4 = 1296\,\text{cis}\left(\frac{8\pi}{3}\right)$

$= 1296\,\text{cis}\left(\frac{2\pi}{3}\right)$

Q11. let $z^3 = 4-4\sqrt{3}i$

$|z^3| = \sqrt{16+16(3)}$

$= 8$

$\theta = \tan^{-1}\left(\frac{4\sqrt{3}}{4}\right)$

$= \frac{\pi}{3}$

$\therefore \arg(z^3) = -\frac{\pi}{3}$.

$\therefore z^3 = 8\,\text{cis}\left(-\frac{\pi}{3}\right)$

$\therefore z_k = 2\,\text{cis}\left(\frac{-\frac{\pi}{3}+2\pi k}{3}\right)$

$\Rightarrow z_1 = 2\,\text{cis}\left(-\frac{\pi}{9}+\frac{2\pi}{3}\right)$

$= 2\,\text{cis}\left(\frac{5\pi}{9}\right)$

$z_2 = 2\,\text{cis}\left(-\frac{\pi}{9}+\frac{4\pi}{3}\right)$

$= 2\,\text{cis}\left(\frac{11\pi}{9}\right)$

$= 2\,\text{cis}\left(\frac{-7\pi}{9}\right)$

$z_3 = 2\,\text{cis}\left(-\frac{\pi}{9}+2\pi\right)$

$= 2\,\text{cis}\left(-\frac{\pi}{9}\right)$

Q12 $z^4 = 16i$

$z^4 = 16\,\text{cis}\left(\frac{\pi}{2}\right)$

$\therefore z_k = \sqrt[4]{16}\,\text{cis}\left(\frac{\frac{\pi}{2}+2\pi k}{4}\right)$

$= 2\,\text{cis}\left(\frac{\pi}{8}+\frac{\pi}{2}k\right)$

$z_1 = 2\,\text{cis}\left(\frac{\pi}{8}+\frac{\pi}{2}\right)$

$= 2\,\text{cis}\left(\frac{5\pi}{8}\right)$

$z_2 = 2\,\text{cis}\left(\frac{\pi}{8}+\pi\right)$

$= 2\,\text{cis}\left(\frac{9\pi}{8}\right)$

$= 2\,\text{cis}\left(-\frac{7\pi}{8}\right)$

$z_3 = 2\,\text{cis}\left(\frac{\pi}{8}+\frac{3\pi}{2}\right)$

$= 2\,\text{cis}\left(\frac{\pi}{8}+\frac{12\pi}{8}\right)$

$= 2\,\text{cis}\left(\frac{13\pi}{8}\right)$

$= 2\,\text{cis}\left(-\frac{3\pi}{8}\right)$

$z_4 = 2\,\text{cis}\left(\frac{\pi}{8}+2\pi\right)$

$= 2\,\text{cis}\left(\frac{\pi}{8}\right)$

Q13. $z^4 = -8\sqrt{2}+8\sqrt{2}i$

$|z^4| = \sqrt{128+128}$

$= 16$

$\theta = \tan^{-1}(1)$

$= \frac{\pi}{4}$

$\therefore \arg(z^4) = \frac{3\pi}{4}$

$\therefore z^4 = 16\,\text{cis}\left(\frac{3\pi}{4}\right)$

$\Rightarrow z_k = 2\,\text{cis}\left(\frac{\frac{3\pi}{4}+2\pi k}{4}\right)$

$= 2\,\text{cis}\left(\frac{3\pi}{16}+\frac{8\pi}{16}k\right)$

$z_1 = 2\,\text{cis}\left(\frac{11\pi}{16}\right)$

$z_2 = 2\,\text{cis}\left(\frac{19\pi}{16}\right)$

$= 2\,\text{cis}\left(-\frac{13\pi}{16}\right)$

$z_3 = 2\,\text{cis}\left(-\frac{5\pi}{16}\right)$

$z_4 = 2\,\text{cis}\left(\frac{3\pi}{16}\right)$

PTO.

Q14 $z^4 + 4 = 0$

$z^4 = -4$

$z^4 = 4\text{cis}(\pi)$

$\therefore z_k = \sqrt[4]{4}\,\text{cis}\left(\frac{\pi + 2\pi k}{4}\right)$

$z_1 = \sqrt[4]{4}\,\text{cis}\left(\frac{3\pi}{4}\right)$

$z_2 = \sqrt[4]{4}\,\text{cis}\left(\frac{5\pi}{4}\right)$

$= \sqrt[4]{4}\,\text{cis}\left(-\frac{3\pi}{4}\right)$

$z_3 = \sqrt[4]{4}\,\text{cis}\left(-\frac{\pi}{4}\right)$

$z_4 = \sqrt[4]{4}\,\text{cis}\left(\frac{\pi}{4}\right)$

Q15 $z_1 = \frac{\sqrt{2} + i\sqrt{6}}{2}$

$= \frac{\sqrt{2}}{2} + i\frac{\sqrt{6}}{2}$

$|z_1| = \sqrt{\frac{1}{2} + \frac{6}{4}}$

$= \sqrt{2}$

$\arg(z_1) = \left(\frac{\sqrt{6}}{2} \div \frac{\sqrt{2}}{2}\right)$

$= \tan^{-1}(\sqrt{3})$

$= \frac{\pi}{3}$

$\therefore z_1 = \sqrt{2}\,\text{cis}\left(\frac{\pi}{3}\right)$

$z_2 = \frac{\sqrt{6} + i\sqrt{2}}{2}$

$= \frac{\sqrt{6}}{2} + i\frac{\sqrt{2}}{2}$

$|z_2| = \sqrt{2}$

$\arg(z_2) = \tan^{-1}\left(\frac{\sqrt{2}}{2} \div \frac{\sqrt{6}}{2}\right)$

$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$= \frac{\pi}{6}$

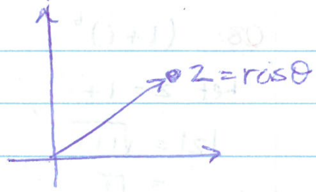
$\therefore z_2 = \sqrt{2}\,\text{cis}\frac{\pi}{6}$

$\therefore \frac{z_1^6 z_2^3}{z_3^4} = \frac{\left(\sqrt{2}\text{cis}\left(\frac{\pi}{3}\right)\right)^6 \left(\sqrt{2}\text{cis}\left(\frac{\pi}{6}\right)\right)^3}{\left(2\text{cis}\left(\frac{\pi}{8}\right)\right)^4}$

$= \frac{8\text{cis}(2\pi)\, 2\sqrt{2}\,\text{cis}\left(\frac{\pi}{2}\right)}{16\text{cis}\left(\frac{\pi}{2}\right)}$

$= \sqrt{2}\,\text{cis}(2\pi) = \sqrt{2}\,\text{cis}(0) = \sqrt{2}$

Q16.



a) $-z = -1 \times r\text{cis}(-\theta)$

$= \text{cis}(\pi) \times r\text{cis}(-\theta)$

$= r\text{cis}(-\theta + \pi)$

$= r\text{cis}(\pi - \theta)$

b) $\frac{1}{z} = \frac{1}{r\text{cis}\theta}$

$= \frac{\text{cis}(-\theta)}{r}$

$= \frac{1}{r}\text{cis}(-\theta)$

c) $-\frac{1}{z} = -\frac{1}{r}\text{cis}(-\theta)$

$= \text{cis}(\pi)\frac{1}{r}\text{cis}(-\theta)$

$= \frac{1}{r}\text{cis}(\pi - \theta)$

d) $-\frac{1}{z^2} = \frac{1}{r}\text{cis}(\pi - \theta) \times \frac{1}{z}$

$= \frac{1}{r}\text{cis}(\pi - \theta)\frac{1}{r}\text{cis}(-\theta)$

$= \frac{1}{r^2}\text{cis}(\pi - 2\theta)$